International Youth Math Challenge

Qualification Round 2022

Problem A

What are the roots of the function $f(x) = (\log(3^x) - 2\log(3)) \cdot (x^2 - 1)$ with $x \in \mathbb{R}$?

Problem B

Find the values of the following infinite sum:

$$1 + \frac{3}{\pi} + \frac{3}{\pi^2} + \frac{3}{\pi^3} + \frac{3}{\pi^4} + \frac{3}{\pi^5} + \cdots$$

Problem C

Determine the numerical value of the following expression without the use of a calculator:

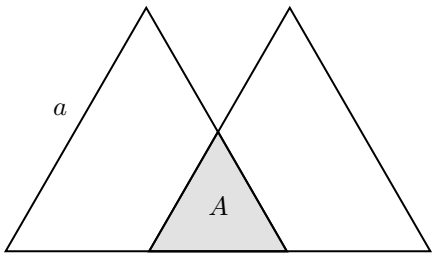
$$\log\left[\ \log(3) \cdot \left(\log(2) \cdot \left(\frac{\sqrt{3} - 2\sin(\pi/3)}{\pi^3 + 1} + 1\right)\right) - \log(2)\log(3) + (-1)^{100}\ \right]$$

Problem D

Let $\sigma(n)$ be the sum of all positive divisors of the integer n and let p be any prime number. Show that $\sigma(n) < 2n$ holds true for all n of the form $n = p^2$.

Problem E

The drawing below shows two equilateral triangles with side length a. The triangles are horizontally shifted by $a/2$. Find the intersection area A of the two triangles (grey area).



Submission Information

To qualify for the next round, you have to solve at least three/four (under/over 18 years) problems correctly. Show your steps! Make sure to submit your solution until *Sunday 16. October 2022 23:59 UTC+0* online! Further information and the submission form is available on the competition website: www.iymc.info