Final Round 2021

The final round exam was given in the form of an online exam. Each participant was given a subset of 20 questions in random order. This paper version is only available for training purposes.

Question 1 : What are the roots of this function?:

$$f(x) = \frac{(1-x)(1+x)}{(2-x)(2+x)}$$

(A) $\{0, 2\}$ (B) $\{-1, 1\}$ (C) $\{0, -1\}$ (D) $\{-2, 2\}$

Question 2 : What is the value of the function $g(x)$ for $x = 0$:

$$g(x) = x^{1+x} + e^x - x^2 - \pi^x$$

(A) 0 (B) 1 (C) 2 (D) π

Question 3 : Let $x = 2$, $y = 4$ and $z = 8$. What is the numerical value of this fraction?:

$$\frac{x^y + y^x}{x \cdot y \cdot z}$$

(A) 0.5 (B) 1 (C) 1.5 (D) 2

Question 4 : How does this sequence of numbers continue?: $4,\ 10,\ 28,\ 82, ...$

(A) 121 (B) 228 (C) 235 (D) 244

Question 5 : What is the value of this limit?:

$$\lim_{x \to 0} \frac{e^x - 1}{x}$$

(A) -1 (B) 0 (C) 1 (D) e

Question 6 : The integer a is called a *quadratic residue* modulo n if an integer b exists such that ...

(A) $b^2 \equiv a \mod n^2$ (B) $b \equiv a \mod n^2$ (C) $b \equiv a^2 \mod n$ (D) $b^2 \equiv a \mod n$

Question 7 : The n-th *Fermat number* has the form ...

(A) $2^n + 1$ (B) $2^{2^n} + 1$ (C) $2^n - 1$ (D) $2^{2^n} - 1$

Question 8 : Find the smallest integer $n > 1$ that is a square and a cube:

(A) 36 (B) 49 (C) 64 (D) 81

Question 9 : Which formula calculates the sum of the first n odd numbers?:

$$1 + 3 + 5 + ... + (2n - 1)$$

(A) n^2 (B) $2n^2 - 1$ (C) $n^2 + n - 1$ (D) $2n^2 + n - 2$

Question 10 : Which x solves this equation?:

$$x^2 - 4x + 4 = -4 + 4x - x^2$$

(A) 0 (B) 1 (C) $\sqrt{2}$ (D) 2

Question 11 : How can you express $\tan\alpha$ with $\cos\alpha$?

(A) $\tan\alpha = \pm\sqrt{1 - \cos^2\alpha}/\cos\alpha$ (B) $\tan\alpha = \pm\sqrt{1 - \cos^2\alpha}/\cos^2\alpha$

(C) $\tan\alpha = \pm\sqrt{1 + \cos^2\alpha}/\cos\alpha$ (D) $\tan\alpha = \pm\sqrt{1 + \cos^2\alpha}/\cos^2\alpha$

Question 12 : Find the smallest value of the function $f(x) = 2^x + |x + 1|$ for $x \in \mathbb{R}$:

(A) 0.25 (B) 0.5 (C) 0.75 (D) 1

Question 13 : Solve this inequality for $x \in \mathbb{R}$:

$$\frac{3x - 3}{3} > \frac{2x + 14}{4}$$

(A) $x > 3$ (B) $x > 5$ (C) $x > 7$ (D) $x > 9$

Question 14 : Find the x that solves this equation:

$$\sin^2 x - \cos^2 x = \frac{1}{2}$$

(A) $x = \pi/2$ (B) $x = \pi/3$ (C) $x = \pi/6$ (D) $x = \pi/12$

Question 15 : Simplify the following fraction:

$$\frac{155551}{1551}$$

(A) $\frac{10101}{101}$ (B) $\frac{13131}{131}$ (C) $\frac{14141}{141}$ (D) $\frac{15151}{151}$

Question 16 : You throw three dice at the same time. What is the probability of getting the numbers 1, 2, and 3?

(A) 1/6 (B) 1/36 (C) 1/64 (D) 1/72

Question 17 : What is the value of this expression?:

$$\frac{\sin(\pi)+\cos(\pi)+\tan(\pi)}{\sin(-\pi)+\cos(-\pi)+\tan(-\pi)}$$

(A) -1 (B) 0 (C) 1/2 (D) 1

Question 18 : The volume of a pyramid with base area B and height h is equal to ...

(A) $Bh/3$ (B) $Bh/2$ (C) $2Bh/3$ (D) Bh

Question 19 : What is the numerical value of this sum?:

$$\sum_{n=1}^{5} \frac{n+n^2}{n}$$

(A) 15 (B) 20 (C) 25 (D) 30

Question 20 : Find the derivative $f'(x)$ of this function:

$$f(x) = x \cdot \ln\left(\frac{1}{x}\right)$$

(A) $\ln\left(\frac{1}{x}\right) - x$ (B) $\ln(x) - x$ (C) $\ln\left(\frac{1}{x}\right) - 1$ (D) $\ln(x) - 1$

Question 21 : Select the number that has exactly eight digits:

(A) $(10)^5$ (B) $(20)^5$ (C) $(30)^5$ (D) $(40)^5$

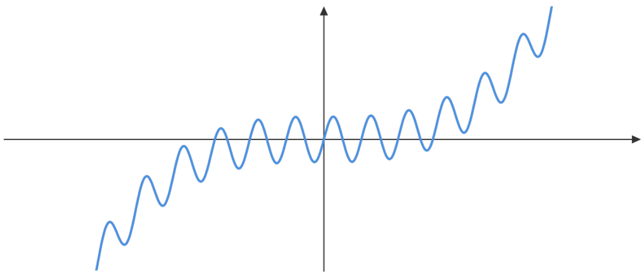
Question 22 : Which one of the following numbers is a perfect square?

(A) 12021 (B) 12121 (C) 12221 (D) 12321

Question 23 : Which number does *not* fit into this sequence: 14, 21, 28, 31, 35, 42, ...

(A) 21 (B) 28 (C) 31 (D) 35

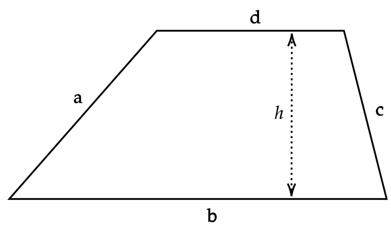
Question 24 : Find the function $f(x)$ with this graph:



(A) $f(x) = x^3/10 + \sin(10x)$ (B) $f(x) = x^3/10 + \cos(10x)$

(C) $f(x) = x^3/10 - \sin(10x)$ (D) $f(x) = x^3/10 - \cos(10x)$

Question 25 : The trapezium below has the sides a, b, c, d, and height h. What is the surface area of this trapezium?



(A) $\frac{1}{2} \cdot (a+b) \cdot h$ (B) $\frac{1}{2} \cdot (a+c) \cdot h$ (C) $\frac{1}{2} \cdot (b+c) \cdot h$ (D) $\frac{1}{2} \cdot (b+d) \cdot h$

Question 26 : What is the value of this sum?:

$$0.1 + (0.1)^2 + (0.1)^3 + (0.1)^4 + (0.1)^5$$

(A) 0.110111 (B) 0.111011 (C) 0.111101 (D) 0.111110