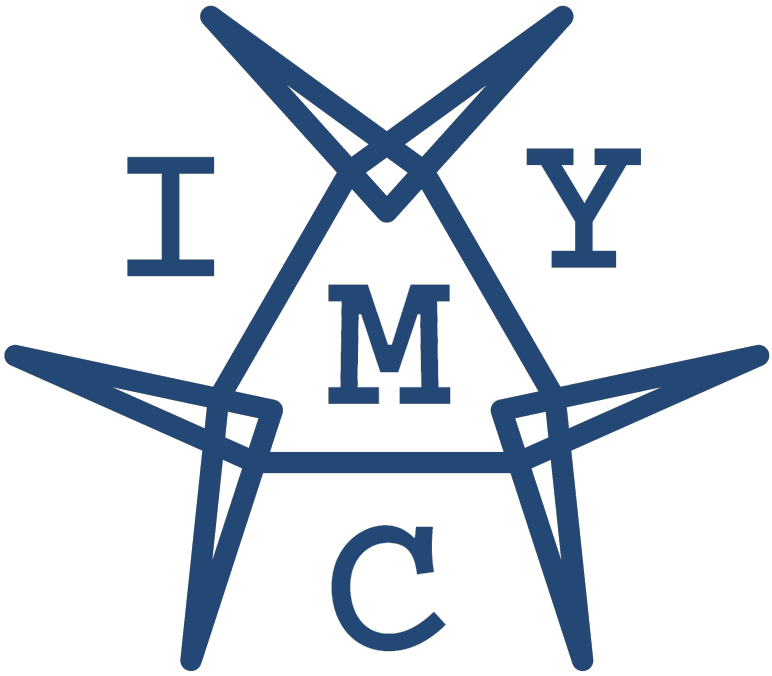


Final Round 2020

The final round exam was given in the form of an online exam. Each participant was given a subset of 30 questions in random order. This paper version is only available for training purposes.

Question 1 : What are the roots of this function?:

$$f(x) = (x+1)^3(x-e^x)^2$$

(A) $\{0, 1\}$ (B) $\{0, -1\}$ (C) $\{1\}$ (D) $\{-1\}$

Question 2 : Let $a = 5$, $b = 10$ and $c = 15$. What is the numerical value of this fraction:

$$\frac{6a + 3c}{a + 2b}$$

(A) 2 (B) 3 (C) 4 (D) 5

Question 3 : How does this sequence of numbers continue?: $4,\ 24,\ 124,\ 624, ...$

(A) 1224 (B) 2224 (C) 2624 (D) 3124

Question 4 : What is the value of this limit?:

$$\lim_{x \to 0} \frac{\sin(x^2)}{x^2}$$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) ∞

Question 5 : Determine the value of the following term:

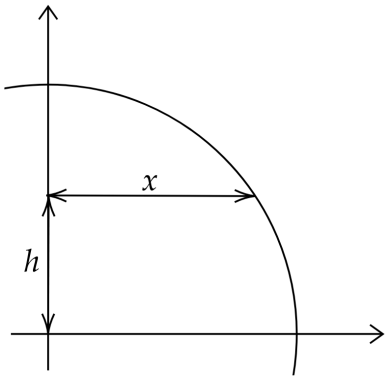
$$\sin(2\pi) + \cos(6\pi) + \tan(8\pi)$$

(A) 1 (B) $\frac{\sqrt{2}}{2}$ (C) 0 (D) -1

Question 6 : The term $\cos(2\alpha)$ can also be written as ...

(A) $2\cos^2\alpha - 1$ (B) $2\sin^2\alpha - 1$ (C) $2\sin\alpha\cos\alpha$ (D) $\sin^2\alpha + \cos^2\alpha$

Question 7 : A circle with circumference π is placed in the centre of a coordinate system (see below). A horizontal line is placed at height h. Determine the distance x between the y-axis and the circle.



(A) $x = \sqrt{\frac{1}{2} - h^2}$ (B) $x = \sqrt{\frac{1}{4} - h^2}$ (C) $x = \sqrt{\frac{1}{2\pi} - h^2}$ (D) $x = \sqrt{\frac{1}{4\pi} - h^2}$

Question 8 : The surface area of a sphere with radius r is equal to ...

(A) $2\pi r^2$ (B) $4\pi r^2$ (C) $2\pi r^2/3$ (D) $4\pi r^2/3$

Question 9 : Determine the value of x from these three equations:

$$2x + y + z = 0$$

$$x + 2y + z = 1$$

$$x + y + 2z = 1$$

(A) $x = -1/2$ (B) $x = 0$ (C) $x = 1/2$ (D) $x = 1$

Question 10 : What is the value of p?:

$$n = 1 + 2 + 3$$

$$m = n + 2n + 3n$$

$$p = \frac{n + m}{2}$$

(A) 15 (B) 17 (C) 19 (D) 21

Question 11 : Find the smallest k such that the following inequality holds true:

$$2^k + 1000 < 3^k + 100 < 4^k$$

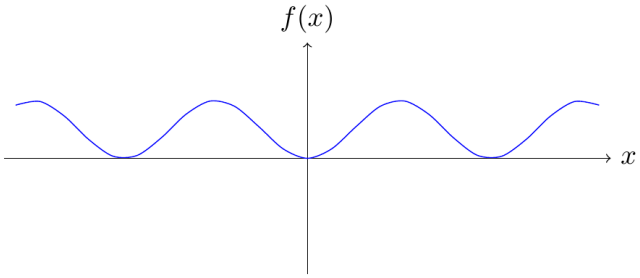
(A) 5 (B) 6 (C) 7 (D) 8

Question 12 : Find the derivative $f'(x)$ of the function $f(x) = \sin\left(\frac{1}{x}\right)$.

(A) $f'(x) = \cos\left(\pi - \frac{1}{x}\right)/x^2$ (B) $f'(x) = \cos\left(\frac{1}{x}\right)/x^2$

(C) $f'(x) = \sin\left(\pi - \frac{1}{x}\right)/x^2$ (D) $f'(x) = \sin\left(\frac{1}{x}\right)/x^2$

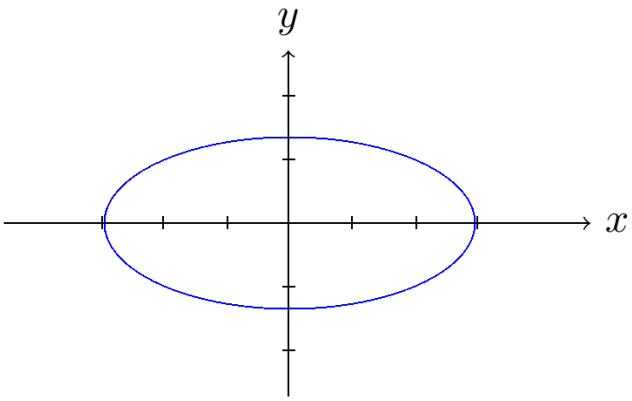
Question 13 : Find the function $f(x)$ with this graph:



(A) $f(x) = 1 + \sin(x)$ (B) $f(x) = 1 - \cos^2(x)$

(C) $f(x) = 1 + \cos(x)$ (D) $f(x) = 1 - \sin(x^2)$

Question 14 : Which equation corresponds to this graph?:



(A) $y^2 = x^2$ (B) $y^2 = 1 - x^2$ (C) $y^2 = 1 - 4x^2$ (D) $4y^2 = 1 - x^2$

Question 15 : What is the value of this infinite sum?:

$$1 + \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \ ...$$

(A) 1/4 (B) 5/4 (C) 25/4 (D) 50/4

Question 16 : What is the correct value for ω such that $\sin(\omega x)$ has a period of 3?

(A) $\omega = \frac{1}{6}\pi$ (B) $\omega = \frac{1}{3}\pi$ (C) $\omega = \frac{2}{3}\pi$ (D) $\omega = \frac{2}{5}\pi$

Question 17 : How does the following term behave for $n \to \infty$?:

$$\left(\sum_{k=1}^{n} \frac{1}{k}\right) - \ln(n)$$

(A) Goes to ∞ (B) Goes to $-\infty$ (C) Goes to a positive constant (D) Goes to a negative constant

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Question 18 : Find the correct $f(x)$ such that this equation holds true:

$$\frac{d}{dx} f(x) = \frac{x}{1+x^2}$$

(A) $\frac{1}{2}\ln(1+x^2)$ (B) $\ln(1+x^2)$ (C) $\frac{x}{2}\ln(1+x^2)$ (D) $x \cdot \ln(1+x^2)$

Question 19 : Determine the value of a_3 in the following expansion:

$$(1+2x)^8 = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ... + a_8 x^8$$

(A) 248 (B) 348 (C) 448 (D) 548

Question 20 : Let F_n be the n-th Fibonacci number. Which one of the following identities is true?

(A) $F_{2n} = F_n^2 + F_{n+2}^2$ (B) $F_{2n+1} = F_n^2 + F_{n+2}^2$

(C) $F_{2n} = F_n^2 + F_{n+1}^2$ (D) $F_{2n+1} = F_n^2 + F_{n+1}^2$

Question 21 : Determine the 4th derivative of the function $f(x) = \cos^2(x)$.

(A) $4\sin^2(x) - 4\cos^2(x)$ (B) $4\cos^2(x) - 4\sin^2(x)$

(C) $8\sin^2(x) - 8\cos^2(x)$ (D) $8\cos^2(x) - 8\sin^2(x)$

Question 22 : The ¡i¿prime number theorem¡/i¿ states that the number of primes less than or equal to N is asymptotically equal to ...

(A) $\ln(N)/N$ (B) $N/\ln(N)$ (C) $N \cdot \ln(N)$ (D) $N \cdot \ln(N) + N$

Question 23 : Which one of the following statements holds true if and only if n is a prime number?

(A) $n|(n-1)! + 1$ (B) $n|(n-1)! - 1$ (C) $n|(n+1)! + 1$ (D) $n|(n+1)! - 1$

Question 24 : Which one is the correct sequence of the ¡i¿harmonic numbers¡/i¿?

(A) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...$ (B) $1, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, ...$

(C) $1, \frac{1}{2}, \frac{5}{6}, \frac{10}{12}, ...$ (D) $1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, ...$

Question 25 : Which term is equal to the following infinite sum for $|x| < 1$?:

$$1 + 2x + 3x^2 + 4x^3 + 5x^4 + \ ...$$

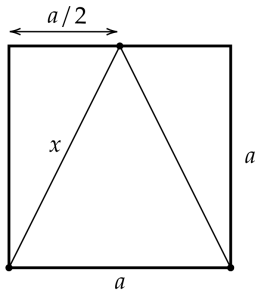
(A) $\frac{1}{1-x}$ (B) $\frac{x}{1-x}$ (C) $\frac{1}{(1-x)^2}$ (D) $\frac{x}{(1-x)^2}$

Question 26 : Which one of the following series is equal to $\ln(1+x)$?

(A) $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} + ...$ (B) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - ...$

(C) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + ...$ (D) $x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \frac{x^5}{5!} - ...$

Question 27 : A triangle is placed into a square with sides of length a as seen below. What is the length x of one of the sides of the triangle?



(A) $x = \sqrt{2}a/2$ (B) $x = \sqrt{3}a/2$ (C) $x = \sqrt{4}a/2$ (D) $x = \sqrt{5}a/2$

Question 28 : You throw two dice with six sides each at the same time. What is the probability of getting a total sum of 4?

(A) $\approx 5\%$ (B) $\approx 8\%$ (C) $\approx 12\%$ (D) $\approx 16\%$

Question 29 : Every positive integer can be written as the sum of at least ...

(A) two integer squares (B) three integer squares

(C) four integer squares (D) five integer squares

Question 30 : The two functions $f(x) = 1 - \sin^2(\pi x)$ and $g(x) = \cos^2(x^2 - \pi^2)$ have an intersection point for ...

(A) $x = \frac{\pi}{2}(1 + \sqrt{4})$ (B) $x = \frac{\pi^2}{2}(1 + \sqrt{4})$ (C) $x = \frac{\pi}{2}(1 + \sqrt{5})$ (D) $x = \frac{\pi^2}{2}(1 + \sqrt{5})$

Question 31 : What is the last digit of this number?:

$$(11^8 \cdot 19 \cdot 23^2)^5$$

(A) 1 (B) 3 (C) 7 (D) 9

Question 32 : Let A and B be two sets of real numbers. Which one of the following identities is true?

(A) $A = (A \cap \varnothing) \cap (A \cup B)$ (B) $A = (A \cap \varnothing) \cup (A \cup B)$

(C) $A = (A \cup \varnothing) \cap (A \cup B)$ (D) $A = (A \cup \varnothing) \cup (A \cup B)$

Question 33 : Find the numerical value of this expression:

$$1 - \log_3 \left(\prod_{k=1}^{100} \frac{1}{27^k} \right)$$

(A) 10100 (B) 10101 (C) 15150 (D) 15151

Question 34 : The expression $\sqrt{6 + 2\sqrt{5}}$ is equal to ...

(A) $1 + \sqrt{5}$ (B) $1 + 2\sqrt{5}$ (C) $\sqrt{3} + \sqrt{5}$ (D) $\sqrt{3} + 2\sqrt{5}$

Question 35 : Which inequality holds for this number?:

$$x = \sqrt{1 + \sqrt{2 + \sqrt{3 + \sqrt{4 + \sqrt{5}}}}}$$

(A) $1 \leq x < 2$ (B) $2 \leq x < 3$ (C) $3 \leq x < 4$ (D) $4 \leq x < 5$

Question 36 : Let a, b, c be the solutions to this equation:

$$0 = x^3 - (2 + 2^2 + 2^3)x^2 + (2^3 + 2^4 + 2^5)x - 2^6$$

What is the value of $a \cdot b \cdot c$?

(A) 2^4 (B) 2^6 (C) 2^8 (D) 2^{10}