# International Youth Math Challenge

## Qualification Round 2018

## Problem A:

Solution:

$$f(x) = (e^x - e^\pi)(e^x - \pi) = 0$$

It can be split into two equations:

$$e^x - e^\pi = 0 \qquad\qquad e^x - \pi = 0$$

$$e^x = e^\pi \qquad\qquad e^x = \pi$$

$$x = \pi \qquad\qquad x = \ln(\pi)$$

So, $\boxed{x = \pi,\ \ln(\pi)}$

## Problem B:

Solution:

$$n^4 - n^3 + n^2 - n \qquad (n \in \mathbb{N})$$

$$= n^2(n^2 - n) + (n^2 - n)$$

$$= (n^2 - n)(n^2 + 1)$$

$$= n(n-1)(n^2+1)$$

If $n$ is even, $n(n-1)(n^2+1)$ is also even since it has $n$ as a factor.

If $n$ is odd, $(n-1)$ and $(n^2+1)$ are even and so, $n(n-1)(n^2+1)$ is also even.

Hence, for all positive integers $n$, $n^4 - n^3 + n^2 - n$ is divisible by 2.

Problem C:

Solution:

The volume of a sphere is given by $\frac{4}{3}\pi r^3$, where $r$ is the radius.

So,

$$\frac{4}{3}\pi r^3 = \pi^3$$

$$r^3 = \frac{3}{4}\pi^2$$

$$\boxed{r = \sqrt[3]{\frac{3}{4}\pi^2}}$$

It is not possible to build such a sphere in reality. Since $\pi$ is a transcendental number, its cube root is impossible to build in reality even if $\pi^2$ is possible.

Problem D:

Solution:

$$\log_2(2^2+5\cdot 2^2\cdot 3)\cdot\left(2\log_3 2+\log_3\left(7-\frac{1}{4}\right)\right)+\frac{(\log_2 128-2)^3}{3+2}+(-1)^{32+\pi}$$

$$=\log_2(2^2(1+15))\cdot\left(\log_3(2^2)+\log_3\left(\frac{27}{4}\right)\right)+\left(\frac{(\log_2(2^7)-2)^3}{5}\right)+(-1)^{32+1}$$

$$=\log_2(2^2\cdot 16)\cdot\left(\cancel{\log_3 2^2}+\log_3 27-\cancel{\log_3 4}\right)+\frac{\cancel{(7-2)^3}\rightarrow 5^3}{5}+(-1)^{33}$$

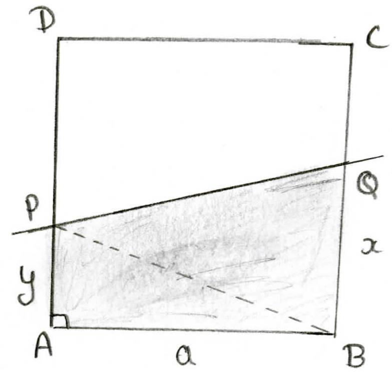
$$=\log_2(2^6)\cdot\log_3(3^3)+25+(-1)$$

$$=6\cdot 3+25-1$$

$$=\boxed{42}$$

Problem E:

Solution:




To simplify the solution, we can label the diagram as above and draw a line segment PB.

The area of a triangle is given by $= \frac{1}{2}$ base $\times$ height

So $\Delta ABP = \frac{1}{2} y a$

and $\Delta PBQ = \frac{1}{2} x a$

The required area, $A(x, y) = \Delta ABP + \Delta PBQ$

$$= \frac{1}{2} y a + \frac{1}{2} x a$$

$$= \boxed{\frac{1}{2} a (x + y)}$$