Problem A
Find the roots of the function \( f(x) = (2^x - 1) \cdot (x^2 + 2x - 3) \) with \( x \in \mathbb{R} \).

Problem B
Show that \( 3 \cdot 4^n + 51 \) is divisible by 3 and 9 for all positive integers \( n \).

Problem C
Determine the numerical value of the following expression without the use of a calculator:
\[
\sqrt{\sin \left( \frac{\pi + \log_2 (\sqrt{2\pi} \cdot 2\pi)}{2^9 - 2^4 - 2^3} \right)} \cdot \sqrt[3]{2^{4-1} + \log_2 (\log_3 (9^{15}) + \pi^{1+(-1)^{17}} + 1) + \frac{(-1)^5 + (-1)^{27}}{(-1)^{766}}}
\]

Problem D
Prove that the inequality \(|\cos(x)| \geq 1 - \sin^2(x)\) holds true for all \( x \in \mathbb{R} \).

Problem E
The white square in the drawing below is located in the centre of the grey rectangle and has a surface area of \( A \). The width of the rectangle is twice the width of the square. What is the surface of the grey area (without the white square)?

Submission Information
To qualify for the next round, you have to solve at least three/four (under/over 18 years) problems correctly.
Show your steps! Make sure to submit your solution until Sunday 11. October 2020 23:59 UTC+0 online!
Further information and the submission form is available on the competition website: www.iymc.info