

# Final Round 2018

**KEEP SECRET BEFORE AND AFTER FINAL EXAM!**

**Important: Read all the information on this page carefully!**

## General Information

- Please read all questions carefully!
- This exam consists of 30 multiple-choice questions.
- To every question, there are four possible answers: A, B, C and D.
- Only one of the four answer is correct!
- Every correct answer gives you one point.
- There are no negative points for wrong answers.
- You have strictly 60 minutes to solve as many problems as possible.
- If you cannot answer a question: Skip it! The final round consists of more questions than we expect you to answer.
- Write your answers on the *Your-Answers-page* only (see next page)!
- Following notation is used for the questions:
  - $x \in \mathbb{R}$  denotes a real number,  $n \in \mathbb{N}$  denotes a positive integer.
  - $f, g$  denote functions. (The domain and co-domain should follow from the context.)
  - The "roots" of a function  $f$  are those  $x$  such that  $f(x) = 0$ .
  - $\pi = 3.141\dots$  denotes the circle constant and  $e = 2.718\dots$  Euler's number.
- **You are allowed to...**
  - use a pencil/pen for writing.
  - use extra blank papers for personal notes.
- **You are not allowed to...**
  - work more than 60 minutes on this exam.
  - use electronic devices (e.g. internet, calculators).
  - use any source of information (e.g. notes, books).
  - receive help from your supervisor or other students.
- **Cheating Policy:** In addition to the presence and supervision of your supervisor during the examination we have various additional methods to detect cheating: This includes methods to detect time violations as well as to detect the usage of tools (e.g. internet) for cheating. Cheating will result in immediate disqualification!

**Good luck!**

# Your Answers

Your Name:

Please write your answers on this page!

No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10
No. 11	No. 12	No. 13	No. 14	No. 15	No. 16	No. 17	No. 18	No. 19	No. 20
No. 21	No. 22	No. 23	No. 24	No. 25	No. 26	No. 27	No. 28	No. 29	No. 30

Please write A, B, C or D into the boxes to give your answers.

**Question 1 :** What are the roots of the function  $f(x) = x^2 + ax - 2a^2$  with  $a \in \mathbb{R}$ ?

- (A)  $\{a, 2a\}$       (B)  $\{-a, 2a\}$       (C)  $\{a, -2a\}$       (D)  $\{-a, -2a\}$
- 

**Question 2 :** Find the point of intersection  $(x, y)$  of  $f(x) = 2x - 3$  and  $g(x) = -x^3$ .

- (A)  $(1, 1)$       (B)  $(-1, 1)$       (C)  $(1, -1)$       (D)  $(-1, -1)$
- 

**Question 3 :** Solve this equation for  $x \in \mathbb{R}$ :

$$\frac{x-1}{2} - \frac{2x-3}{4} - \frac{3x-4}{5} = 3$$

- (A)  $x = \frac{39}{12}$       (B)  $x = -\frac{39}{12}$       (C)  $x = \frac{79}{24}$       (D)  $x = -\frac{79}{24}$
- 

**Question 4 :** What is the value of  $\cos(7\pi/6)$ ?

- (A)  $1/2$       (B)  $-1/2$       (C)  $\sqrt{3}/2$       (D)  $-\sqrt{3}/2$
- 

**Question 5 :** Find the derivative  $f'(x)$  of the function  $f(x) = x^x$ .

- (A)  $f'(x) = (\ln(x) + 1) \cdot x^x$       (B)  $f'(x) = \ln(x + 1) \cdot x^x$   
(C)  $f'(x) = (\ln(x) + x) \cdot x^x$       (D)  $f'(x) = \ln(x + x^x) \cdot x^x$
- 

**Question 6 :** Find the numerical value of this expression:

$$\sin^2\left(\frac{\pi}{2}\right) \frac{\pi^{2-\cos(2\pi)}}{7+(-1)^2} + \cos^2\left(\frac{\pi}{4} + \frac{\pi}{4}\right) \frac{\pi}{3^2-1}$$

- (A)  $\pi$       (B)  $\pi/2$       (C)  $\pi/4$       (D)  $\pi/8$
- 

**Question 7 :** Find the value of this infinite sum:  $\sum_{n=0}^{\infty} \frac{3^n}{4^n}$

- (A)  $1/3$       (B)  $1/4$       (C)  $3$       (D)  $4$
- 

**Question 8 :** Which one is the set of prime numbers?

- (A)  $\{n \in \mathbb{N} : 1|n \wedge n|n\}$       (B)  $\{n \in \mathbb{N} : 1|n \wedge n|n \wedge n > 1\}$   
(C)  $\{n \in \mathbb{N} : |\{t \in \mathbb{N} : t|n\}| = 2\}$       (D)  $\{n \in \mathbb{N} : |\{t \in \mathbb{N} : t|n \wedge t > 1\}| = 2\}$
-

**Question 9 :** How can you express  $\sin(x)$  with  $\cos(x)$ ?

- (A)  $\sin(x) = \cos(\pi/2 + x)$                       (B)  $\sin(x) = \cos(\pi/2 - x)$   
 (C)  $\sin(x) = \cos(\pi + x)$                       (D)  $\sin(x) = \cos(\pi - x)$
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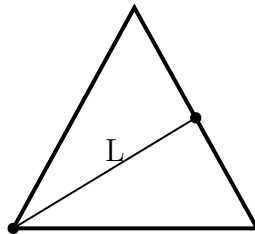
**Question 10 :** Which number divides  $n^3 + 5n$  for all  $n \in \mathbb{N}$ ?

- (A) 4                      (B) 5                      (C) 6                      (D) 7
- 

**Question 11 :** Find the smallest value of the function  $f(x) = x^2 + 2^{x^2}$  for  $x \in \mathbb{R}$ .

- (A) 0                      (B) 1/2                      (C) 1                      (D) 2
- 

**Question 12 :** Below you find an equilateral triangle with all sides having a length of 1. A straight line intersects a corner and the center of the triangle. Find the length  $L$  of this line:



- (A)  $L = 1/2$                       (B)  $L = \sqrt{2}/2$                       (C)  $L = 3/2$                       (D)  $L = \sqrt{3}/2$
- 

**Question 13 :** Solve this inequality for  $x \in \mathbb{R}$ :

$$\frac{5(9-x)}{3} + 1 < 16$$

- (A)  $x > 0$                       (B)  $x > 1$                       (C)  $x > 3$                       (D)  $x > 5$
- 

**Question 14 :** Which equation is equal to  $\cos(\alpha + \beta)$  with  $\alpha, \beta \in \mathbb{R}$ ?

- (A)  $\sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$                       (B)  $\sin(\alpha) \cos(\beta) - \cos(\alpha) \sin(\beta)$   
 (C)  $\cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta)$                       (D)  $\cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$
- 

**Question 15 :** What is the last digit of  $11^5 \cdot 17^3 \cdot 33^3$ ?

- (A) 1                      (B) 2                      (C) 3                      (D) 5
-

**Question 16 :** Which number is also known as *Golden ratio*?

- (A)  $\frac{1+\sqrt{3}}{2}$       (B)  $\frac{1-\sqrt{3}}{2}$       (C)  $\frac{1+\sqrt{5}}{2}$       (D)  $\frac{1-\sqrt{5}}{2}$
- 

**Question 17 :** Find the value of  $x$  that is a solution to  $\sqrt{2} = 4 \sin(x) \cos(x)$ .

- (A)  $x = \pi$       (B)  $x = \pi/2$       (C)  $x = \pi/4$       (D)  $x = \pi/8$
- 

**Question 18 :** Find the correct inequality for  $a = \frac{1001}{1002}$ ,  $b = \frac{1003}{1004}$  and  $c = \frac{1002}{1003}$ :

- (A)  $a < b < c$       (B)  $a < c < b$       (C)  $c < a < b$       (D)  $c < b < a$
- 

**Question 19 :** Find the correct inequality with  $x = \pi^{\pi^2}$ :

- (A)  $27^2 \leq x \leq 3^6$       (B)  $3^6 \leq x \leq 3^9$       (C)  $3^6 \leq x \leq 9^6$       (D)  $27^2 \leq x \leq 3^9$
- 

**Question 20 :** What are the roots  $x \in \mathbb{R}$  of the function  $f(x) = \frac{x^2-1}{x+1}$ ?

- (A)  $\{1, -1\}$       (B)  $\{1, -\sqrt{2}\}$       (C)  $\{-1, \sqrt{2}\}$       (D)  $\{-1, -\sqrt{2}\}$
- 

**Question 21 :** Find the solutions  $x \in \mathbb{R}$  to this equation:

$$\pi \sin^2(x^3) - x^2 \cos^2(x^3) = x^2 \sin^2(x^3) - \pi \cos^2(x^3)$$

- (A)  $\{1, -1\}$       (B)  $\{\sqrt{\pi}, -\sqrt{\pi}\}$       (C)  $\{\pi, -\pi\}$       (D)  $\{\pi^2, -\pi^2\}$
- 

**Question 22 :** Find an expression for the product of the first  $k$  odd numbers:

$$P = 1 \cdot 3 \cdot 5 \cdot \dots \cdot (2k - 1)$$

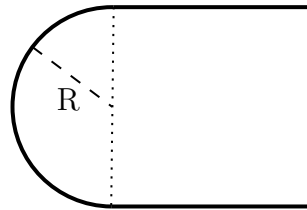
- (A)  $P = \frac{(2k)!}{2^k}$       (B)  $P = \frac{(2k)!}{2^k k!}$       (C)  $P = \frac{(2k+1)!}{2^k}$       (D)  $P = \frac{(2k+1)!}{2^k k!}$
- 

**Question 23 :** Find the sum of the first 100 numbers:

$$S = 1 + 2 + 3 + \dots + 99 + 100$$

- (A) 4950      (B) 5050      (C) 5150      (D) 5380
-

**Question 24 :** A semicircle with radius  $R$  is connected to a square (see below). Find the perimeter of this shape (length of bold line):



- (A)  $(3 + \pi)R$       (B)  $(3 + 2\pi)R$       (C)  $(6 + \pi)R$       (D)  $(6 + 2\pi)R$

**Question 25 :** Which equation is true for any triangle with angles  $\alpha, \beta, \gamma$  and sides  $a, b, c$ ?

- (A)  $c^2 = a^2 + b^2 + 2ab \sin \gamma$       (B)  $c^2 = a^2 + b^2 + 2ab \cos \gamma$   
 (C)  $c^2 = a^2 + b^2 - 2ab \sin \gamma$       (D)  $c^2 = a^2 + b^2 - 2ab \cos \gamma$

**Question 26 :** What is the value of this division:  $\frac{2018^{2018}}{2018}$

- (A) 101      (B) 1001      (C) 10001      (D) 100001

**Question 27 :** What is the value of  $\frac{2x-3}{x-1}$  for  $x \rightarrow \infty$ ?

- (A) 0      (B) 1      (C) 2      (D) 3

**Question 28 :** How many last digits of  $17!$  are zeros?

- (A) 2      (B) 3      (C) 4      (D) 5

**Question 29 :** What is the value of  $S = \sum_{n=0}^{\infty} \frac{1}{n^2}$ ?

- (A)  $1/2$       (B)  $\pi/3$       (C)  $\pi^2/6$       (D)  $\pi^3/9$

**Question 30 :** How likely have two of 23 students birthday on the same day?

- (A)  $\approx 10\%$       (B)  $\approx 30\%$       (C)  $\approx 50\%$       (D)  $\approx 70\%$