Final Round 2022

The final round exam was given in the form of an online exam. Each participant was given a subset of 20 questions in random order. This paper version is only available for training purposes.

Question 1 : What are the roots of this function?:

$$f(x) = \log_3(9^x) \cdot (x^2 - 4)$$

(A) $\{-1, 0, 1\}$ (B) $\{-2, 0, 2\}$ (C) $\{-1, 0.5, 1\}$ (D) $\{-2, 0.5, 2\}$

Question 2 : What is the value of this infinite sum?:

$$1 + \frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \frac{2}{3^4} + \ldots$$

(A) 2   (B) 3   (C) 1/3   (D) 3/2

Question 3 : What is the value of this infinite sum?:

$$\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{2^3} - \frac{1}{3^3}\right) + \ldots$$

(A) 1/2   (B) 1/3   (C) 2/3   (D) 3/2

Question 4 : Find the numerical value of this expression:

$$\log_3\left(5 \cdot \log_2(8 + 2^3) + 7\right)$$

(A) 1   (B) 2   (C) 3   (D) 4

Question 5 : Let $\sigma(n)$ be the sum of all positive divisors of the integer n. Let p be a prime number. It is true that ...

(A) $\sigma(p^2) = 2p + 1$ (B) $\sigma(p^2) > 2p^2$ (C) $\sigma(p^2) = 2p^2$ (D) $\sigma(p^2) < 2p^2$

Question 6 : Let $\sigma(n)$ be the sum of all positive divisors of the integer n. The value of $\sigma(30)$ is ...

(A) 27   (B) 38   (C) 42   (D) 72

Question 7 : The area of an equilateral triangle with side length a is equal to ...

(A) $a^2 \cdot \sqrt{2}/3$ (B) $a^2 \cdot \sqrt{2}/4$ (C) $a^2 \cdot \sqrt{3}/3$ (D) $a^2 \cdot \sqrt{3}/4$

Question 8 : Determine A such that the function $f(x)$ intersects the point $(2, 2)$:

$$f(x) = Ax^2 + x + 1$$

(A) $A = 0$ (B) $A = -1$ (C) $A = -1/2$ (D) $A = -1/4$

Question 9 : Find a point of intersection of the functions $f(x)$ and $g(x)$:

$$f(x) = 2x^2 + 4$$
$$g(x) = 3x^2 + 3$$

(A) $(-1, -4)$ (B) $(1, 4)$ (C) $(-1, -6)$ (D) $(1, 6)$

Question 10 : Find the derivative $f'(x)$ of this function:

$$f(x) = \pi^{\sin(x)}$$

(A) $f'(x) = \sin(x) \cdot \ln(\pi) \cdot \pi^{\sin(x)}$ (B) $f'(x) = \cos(x) \cdot \ln(\pi) \cdot \pi^{\sin(x)}$

(C) $f'(x) = \sin(x) \cdot \ln(\pi) \cdot \pi^{\cos(x)}$ (D) $f'(x) = \cos(x) \cdot \ln(\pi) \cdot \pi^{\cos(x)}$

Question 11 : Let H_n define the sum of reciprocals of all integers from 1 to n. Which one of the following identities is correct?

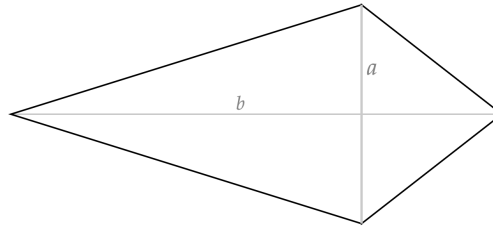(A) $1 = n \cdot (H_{2n} - H_{2n-1})$ (B) $1 = n \cdot (H_{2n} - H_{2n-2})$

(C) $1 = 2n \cdot (H_{2n} - H_{2n-1})$ (D) $1 = 2n \cdot (H_{2n} - H_{2n-2})$

Question 12 : What is the value of φ such that this equation holds?:

$$(1 + 2 + 3 + 4)^2 = 1^2 + 2^2 + 3^2 + 4^2 + \varphi$$

(A) $\varphi = 15$ (B) $\varphi = 30$ (C) $\varphi = 50$ (D) $\varphi = 70$

Question 13 : What is the surface area of a kite with diagonals a and b?:

(A) $ab/4$ (B) $ab/2$ (C) $3ab/4$ (D) $3ab/2$

Question 14 : Let $\{x\}$ be the fractional part of x. Given the following graph, find the function $f(x)$:
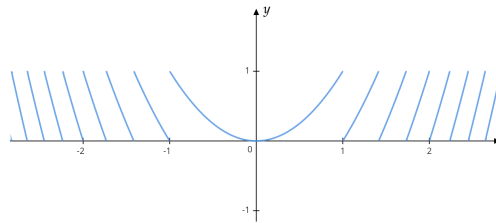
(A) $f(x) = \{x^2\} - x$ (B) $f(x) = \{x^2\}$

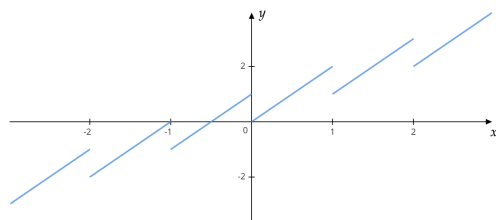(C) $f(x) = \{x^3\} - x$ (D) $f(x) = \{x^3\}$

Question 15 : Let $\{x\}$ be the fractional part of x. Given the following graph, find the function $f(x)$:

(A) $f(x) = \{x/2\} + x$ (B) $f(x) = \{x/2\} - x$

(C) $f(x) = \{x\} + x$ (D) $f(x) = \{x\} - x$

Question 16 : Let $\{x\}$ be the fractional part of x and $\gamma = 0.577...$ a constant. Which function $f(x)$ makes this equation true?:

$$\int_1^\infty f(x)dx = 1 - \gamma$$

(A) $f(x) = \{x\}/x$

(B) $f(x) = \{x^2\}/x$

(C) $f(x) = \{x\}/x^2$

(D) $f(x) = \{x^2\}/x^2$

Question 17 : The integer p is a *twin prime* if ...

(A) $p+1$ or $p-1$ is prime.

(B) $p+2$ or $p-2$ is prime.

(C) $2p+1$ or $2p-1$ is prime.

(D) $2p+2$ or $2p-2$ is prime.

Question 18 : The integer p is a *Germain prime* if ...

(A) $p+1$ is prime.　　(B) $p+2$ is prime.　　(C) $2p+1$ is prime.　　(D) $2p+2$ is prime.

Question 19 : Let $\pi_{a,b}(x)$ be the number of primes $p \le x$ for which $ap+b$ is also prime. What is the value of $\pi_{1,2}(10)$?

(A) $\pi_{1,2}(10) = 2$　　(B) $\pi_{1,2}(10) = 3$　　(C) $\pi_{1,2}(10) = 4$　　(D) $\pi_{1,2}(10) = 5$

Question 20 : How does this sequence of numbers continue?:

$$1, \ 2, \ 7, \ 8, \ 13, \ 14, \ 19, ...$$

(A) 20　　　　　　(B) 21　　　　　　(C) 22　　　　　　(D) 23

Question 21 : Let H_n define the sum of reciprocals of all integers from 1 to n. What is the numerical value of $H_{10} - H_7$?

(A) $H_{10} - H_7 = \frac{121}{360}$

(B) $H_{10} - H_7 = \frac{123}{360}$

(C) $H_{10} - H_7 = \frac{127}{360}$

(D) $H_{10} - H_7 = \frac{129}{360}$

Question 22 : The expression $(2 + \sqrt{3})^2 \cdot (2 - \sqrt{3})^2$ is equal to ...

(A) 1　　　　　　(B) 2　　　　　　(C) 3　　　　　　(D) 6

Question 23 : Let $n! = 1 \cdot 2 \cdot ... \cdot 10$. Which inequality holds true for $\ln(10!)$?

(A) $9 \leq \ln(10!) < 12$ (B) $12 \leq \ln(10!) < 15$

(C) $15 \leq \ln(10!) < 18$ (D) $18 \leq \ln(10!) < 21$

Question 24 : Find the correct $f(x)$ such that this equation holds true:

$$f'(x) = \frac{\cos(x) - 2\sin(x)/x}{x^2}$$

(A) $f(x) = \frac{\sin(x)}{x}$ (B) $f(x) = \frac{\cos(x)}{x}$ (C) $f(x) = \frac{\cos(x)}{x^2}$ (D) $f(x) = \frac{\sin(x)}{x^2}$

Question 25 : Determine the value of C:

$$C = A^B + B^A - C$$
$$B = A^2 + 2^A$$
$$A = 3^2 - 2^3$$

(A) 1   (B) 2   (C) 3   (D) 4

Question 26 : Determine the value of the following term:

$$\sin\left(\pi \cdot \cos\left(\pi \cdot \frac{\tan(\pi/6)}{\sqrt{3}}\right)\right)$$

(A) 0   (B) 1   (C) $\sqrt{2}/2$ (D) $\sqrt{3}/2$

Question 27 : You throw a dice (with the numbers 1 to 6) twice. What is the probability of never throwing the number 6?

(A) $\approx 50\%$ (B) $\approx 60\%$ (C) $\approx 70\%$ (D) $\approx 80\%$