International Youth Math Challenge

Qualification Round 2021 - Solution

Problem A

$a_7 = 37$, $a_n = \sum_{n=2}^{n+1} k = \frac{n(n+3)}{2}$

$b_7 = 20160$, $b_n = \frac{(n+1)!}{2}$

Problem B

$x_1 = 0$
$x_2 = \frac{1}{2} + \sqrt{\frac{7}{12}} = \frac{3+\sqrt{21}}{6} \approx 1.26$
$x_3 = \frac{1}{2} - \sqrt{\frac{7}{12}} = \frac{3-\sqrt{21}}{6} \approx -0.26$

Problem C

Using basic calculation rules: $(3+0) \cdot 1 = 3$

Problem D

It is $(n+2) \cdot \sin(n) \leq n+2$. Thus, it is sufficient to show $2^{n+1} > n+2$. We use induction:

1. Check $n = 1$: $4 > 3$
2. Assume $n = k$ holds: $2^{k+1} > k+2$
3. Show that $n = k+1$ holds: $2^{n+2} = 2^{n+1} \cdot 2 = 2^{n+1} + 2^{n+1} > k+2+1$
   This is true as $n > -1 \quad \Rightarrow \quad 2^{n+1} > 1$.

Problem E

From $\frac{x\cos(\alpha)}{l} = \frac{z}{y+l}$ and $\sin\alpha = \frac{l}{x}$ we get:

$$z = (x\sin\alpha + y) \cdot \frac{\cos(\alpha)}{\sin(\alpha)} = x\cos(\alpha) + y\cot(\alpha)$$