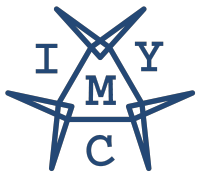
International Youth Math Challenge

Training and Problems

www.iymc.info

Version: V.08.2020



Problem: 2018-QR-A

Problem

Find the roots of

$$f(x) = (e^x - e^\pi)(e^x - \pi)$$

where e denotes Euler's number.

Problem: 2019-QR-A

Problem

Find the maximum value of the function

$$f(x) = x + x^2 - x^3$$

for $x \geq 0$.

Problem: 2018-QR-B

Problem

Show that $n^4 - n^3 + n^2 - n$ is divisible by 2 for all positive integers n.

Problem: 2019-QR-D

Problem

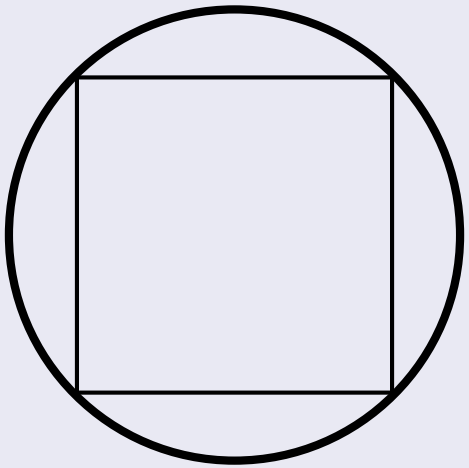
You have given following three equations below with $\alpha, \beta, \gamma \in \mathbb{R}$. What is the value of α?

$$\alpha + \beta + \gamma = 1$$
$$\beta + \gamma + \beta = 1$$
$$\gamma + \beta + \gamma = 1$$

Problem: 2019-QR-E

Problem

The circle in the drawing below has a surface area of $A_1 = 1m^2$. Determine the surface area A_2 of the square that was placed inside of the circle.



Problem: 2019-PF-A1

Problem

Find the area enclosed by these three functions:

$$f(x) = 1, \quad g(x) = x + 1, \quad h(x) = 9 - x$$

Problem: 2019-PF-A2

Problem

Find the roots of this function:

$$f(x) = 3^x \cdot (\log_2(x) - 3)^5 \cdot e^{x^2-3x}$$

Problem: 2018-PF-A5

Problem

Find all x such that $|x^2 - 1| < 2x$.

Problem: 2019-PF-B2

Problem

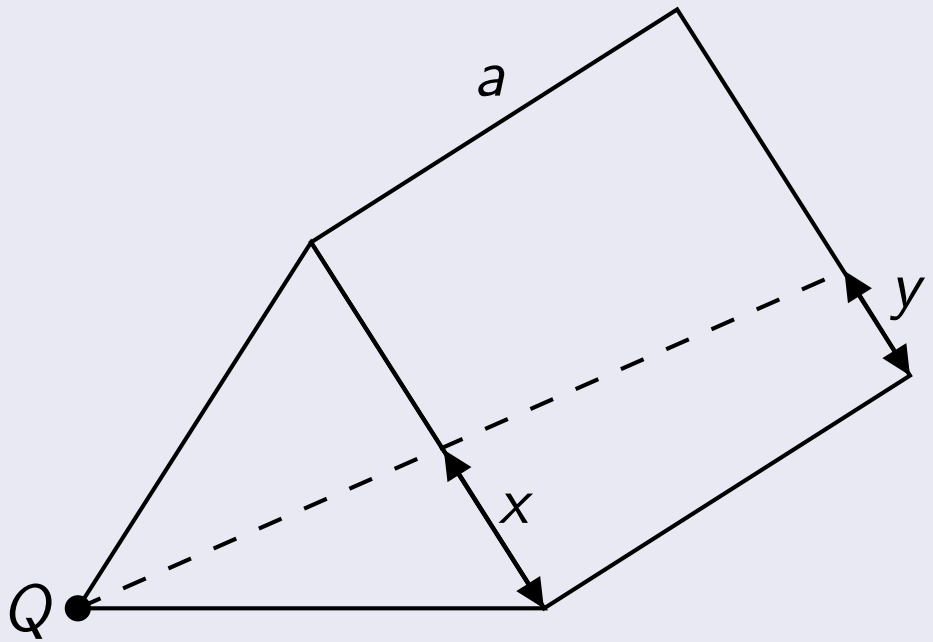
Prove the following inequality between the harmonic, geometric, and arithmetic mean with $x, y \geq 0$:

$$\frac{2}{\frac{1}{x} + \frac{1}{y}} \leq \sqrt{xy} \leq \frac{x+y}{2}$$

Problem: 2019-PF-B4

Problem

Consider an equal-sided triangle connected to a square with side a (see drawing). A straight line from Q intersects the square at x and y. You have given x, find an equation for the intersection at $y(x)$.



Problem: 2018-PF-B1

Problem

Show that $2^{3n} - 1$ is divisible by 7 for all positive integers n.

Problem: 2018-PF-B3

Problem

Find the value of this infinite sum: $\sum_{n=0}^{\infty} \frac{2^{2n} + 2^n}{2^{3n}}$.

Problem: 2018-PF-B4

Problem

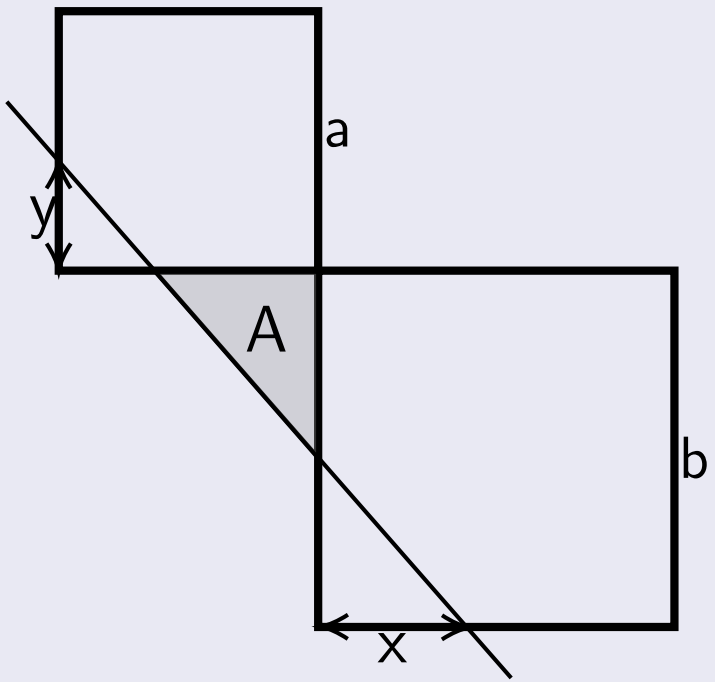
Give a closed expression for the function $g(n)$ with the following behaviour:

$$g(n) = \begin{cases} 0, & n \text{ even} \\ n, & n \text{ odd} \end{cases}$$

Problem: 2018-PF-B6

Problem

The drawing below shows two squares with side a and b. A straight line intersects the squares at y and x. Calculate the gray area $A(a, b, x, y)$ between the squares and the line.



Problem: 2019-PF-C1

Problem

The sum of divisor function $\sigma(n)$ returns the sum of all divisors d of the number n:

$$\sigma(n) = \sum_{d|n} d$$

We denote N_k any number that fulfils the following condition:

$$\sigma(N_k) \geq k \cdot N_k$$

Find examples for N_3, N_4, N_5 and prove that they fulfil this condition.

Problem: 2019-F-1

Problem

What are the roots of the function $f(x) = \frac{x^2-4x+3}{2^x-4}$?

(A) $\{1, 3\}$ (B) $\{1, 4\}$ (C) $\{-1, 3\}$ (D) $\{-1, 4\}$

Problem: 2019-F-3

Problem

How does this sequence of numbers continue?:

$$7,\ 26,\ 63,\ 124, \ldots$$

(A) 205 (B) 215 (C) 225 (D) 235

Problem: 2019-F-4

Problem

What is the value of $\sin(150^\circ) + \cos(4\pi/3)$?

(A) $-1/2$ (B) 0 (C) $1/2$ (D) 1

Problem: 2019-F-5

Problem

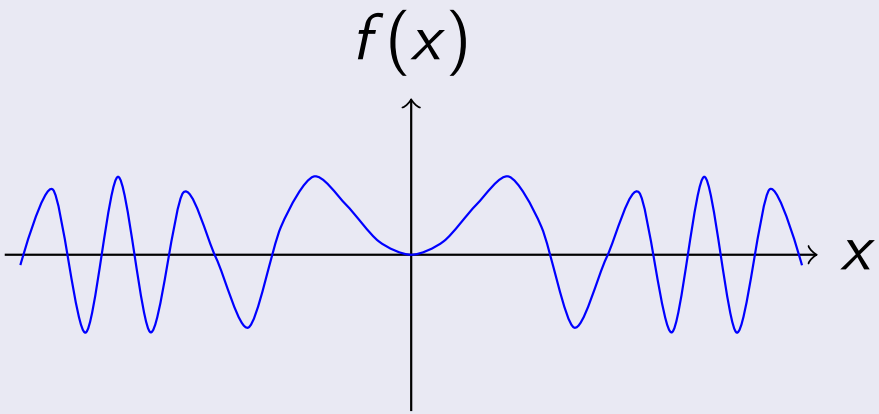
Find the result of this division: $\frac{111111}{11}$

(A) 10001 (B) 10101 (C) 10110 (D) 11111

Problem: 2019-F-11

Problem

Find the function $f(x)$ with this graph:



(A) $f(x) = \sin(x^2)$ (B) $f(x) = \sin^2(x)$

(C) $f(x) = \sin^2(x^2)$ (D) $f(x) = \sin(1/x)$

Problem: 2019-F-13

Problem

Determine the value of this alternating sum:

$$\sum_{n=1}^{1550} (-1)^n \cdot n$$

(A) 225 (B) 775 (C) 1549 (D) 1550

Problem: 2019-F-15

Problem

What are the roots of this function?

$$f(x) = \pi^3 - (\pi + \pi^2 + \pi^3)x + (1 + \pi + \pi^2)x^2 - x^3$$

(A) $\{1, \pi, \pi^2\}$ (B) $\{\pi, \pi^2, \pi^3\}$

(C) $\{-1, \pi, \pi^2\}$ (D) $\{-\pi, \pi^2, \pi^3\}$

Problem: 2019-F-17

Problem

For which n is $p_n = n^2 - n + 41$ not a prime number?

(A) 41 (B) 13 (C) 27 (D) 60

Problem: 2019-F-21

Problem

The binary representation of the decimal number 127 is ...

(A) 1111100 (B) 1111101 (C) 1111110 (D) 1111111

Problem: 2019-F-39

Problem

What is the probability to throw a dice six times without getting a six?

(A) $\approx 16\%$ (B) $\approx 33\%$ (C) $\approx 66\%$ (D) $\approx 83\%$

Problem: 2019-F-40

Problem

You have given a triangle with two sides of equal length. Determine the length of the third side given the circumference U to maximize the area of the triangle.

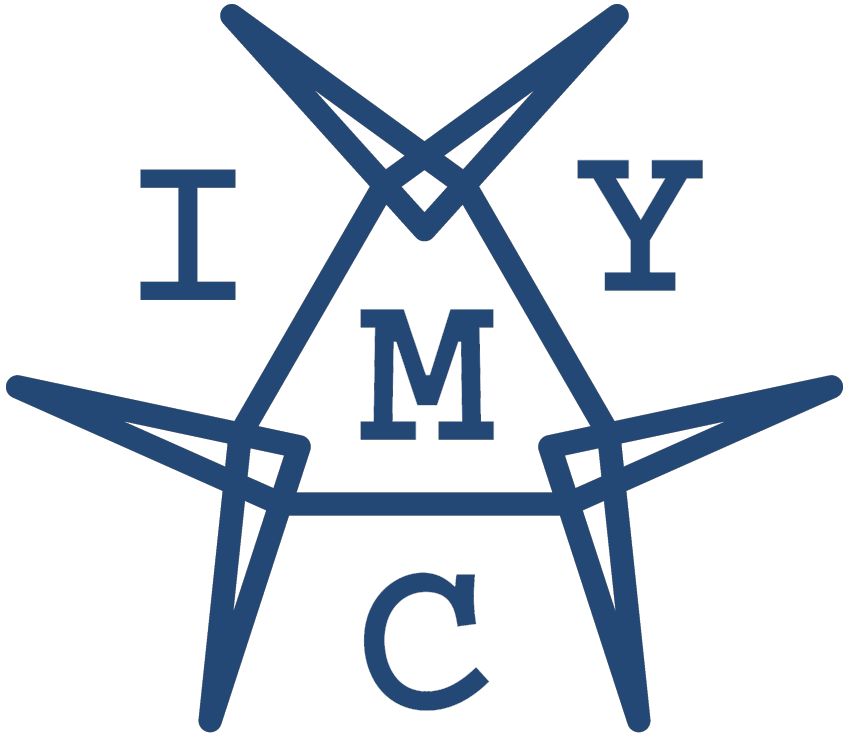
(A) $U/2$ (B) $U/3$ (C) $U/4$ (D) $U/5$

Problem: 2019-F-40

Problem

You have given a triangle with two sides of equal length. Determine the length of the third side given the circumference U to maximize the area of the triangle.

(A) $U/2$ (B) $U/3$ (C) $U/4$ (D) $U/5$



International Youth Math Challenge

www.iymc.info