International Youth Math Challenge

Qualification Round 2021

Problem A

Continue the two sequences of numbers below and find an equation to each of the sequences:

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Equation**
a_n	2	5	9	14	20	27		
b_n	1	3	12	60	360	2520		

Problem B

Find all $x \in \mathbb{R}$ that solve this equation: $123 = x \cdot (2x \cdot (3x - 3) - 2) + 100 + 20 + 3$

Problem C

Determine the numerical value of the following expression without the use of a calculator:

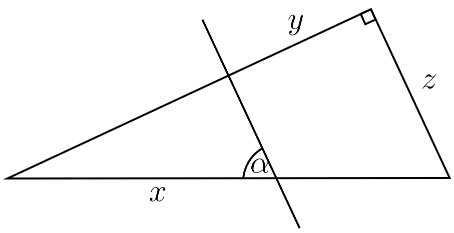
$$\left(\frac{\log_{10}\left(1000^{100}\right)}{100} + \sum_{n=1}^{100} \frac{\sin(\pi n) + 1}{(-1)^n} \right) \cdot \sqrt{\prod_{m=1}^{1000} \frac{1}{\cos(\pi m)^2}}$$

Problem D

Prove that $2^{n+1} > (n+2) \cdot \sin(n)$ for all positive integers n.

Problem E

The drawing below shows a right-angled triangle. A straight line crosses the triangle parallel to the line z and encloses an angle of α. The lengths x and y of the bottom and top line segments as well as the angle α are given. Find an equation for the length z.



Submission Information

To qualify for the next round, you have to solve at least three/four (under/over 18 years) problems correctly. Show your steps! Make sure to submit your solution until *Sunday 17. October 2021 23:59 UTC+0* online! Further information and the submission form is available on the competition website: www.iymc.info